Electron and Vibrational Energy Conservation Equations for Aeroassisted Orbital Transfer Vehicles

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THERE is a renewed interest in thermal and chemical nonequilibrium flows because of the current proposal to develop aeroassisted orbital transfer vehicles (AOTV's).[1] A set of basic governing equations using the continuum and three-temperature assumptions has been derived by Lee[2] for the AOTV flight regimes, where the electron energy may be less than a few electron volts. The conservation equations[3] were derived by accounting for the differences among the translational-rotational T, vibrational T_v, and electron T_e temperatures. It is well known[4-9] that the electron energy loss in molecules (N_2, O_2, etc.) at electron energy quantities of less than a few electron volts is dominated by both the electron-rotation (*eR*) and electron-vibration (*eV*) energy transfers. Therefore, the rate of electron energy loss due to inelastic collisions with molecules $Q_{\mathrm{inela},e}$ in the electron energy conservation equation[2] should include the *eR* energy transfer rate

$$Q_{eR}=\sum_s Q_{eR,s} \tag{1}$$

where $Q_{eR,s}$ is the rate of electron energy loss due to rotational excitations of molecular species s; Q_{eR} is omitted in Ref. 2. The *eV* energy transfer rate

$$Q_{eV}=\sum_s Q_{eV,s} \tag{2}$$

where $Q_{eV,s}$ is the rate of electron energy loss due to vibrational excitations of molecular species s, is taken into account in both the electron and vibrational energy conservation equations of Ref. 2. However, the Landau-Teller type expression for $Q_{eV,s}$ (in Ref. 2) was derived rather artificially and its validity is questionable.

In this Note, an expression for $Q_{eR,s}$ and an alternative expression for $Q_{eV,s}$ are presented. $Q_{eR,s}$ is generally represented[10] by

$$Q_{eR,s}=n_e\sum_i\sum_j \Delta E_{ji}k_{ij}n_i \tag{3}$$

where n_e and n_i are the number densities of electrons and molecules in rotational state i, respectively; ΔE_{ji} ($=E_j-E_i$) the rotational energy (E_i) difference; and k_{ij} the rate coefficient for the rotational transition $i\rightarrow j$, the subscript s being omitted. For the transition $i\rightarrow j$, k_{ij} is related with the collision cross section σ_{ij} by

$$k_{ij}=(1/n_e)\int\sigma_{ij}c_e' f_e \mathrm{d}c_e' \tag{4}$$

where c_e' is the electron random velocity with respect to the mean electron velocity,[2,3] and f_e is the electron velocity distribution function. For N_2 and O_2, σ_{ij} is usually taken as the Gerjuoy-Stein formula.[4-9] Assuming that n_i is a Boltzmann distribution with the translational-rotational temperature T, f_e is a Maxwellian distribution with the electron temperature T_e, and the rotational characteristic temperature $\theta_{R,s}\ll T$, T_e, $Q_{eR,s}$ is reduced to the Landau-Teller type expression[5]

$$Q_{eR,s}=n_e\left(\frac{3}{2}\kappa T_e-\frac{3}{2}\kappa T\right)\Big/\tau_{eR,s} \tag{5}$$

with the *eR* relaxation time

$$\tau_{eR,s}=\left(\frac{3}{16}\right)\frac{(\pi m_e T_e/2\kappa)^{1/2}}{n_s\theta_{R,s}\sigma_{R,s}} \tag{6}$$

where κ is the Boltzmann constant; m_e the electron mass; n_s ($=\Sigma_i n_i$) the molecular number density; and $\sigma_{R,s}=8\pi Q_s^2 a_0^2 \div 15$ the rotational cross section in which Q_s is the effective quadrupole moment in units of ea_0^2, a_0 the Bohr radius, and e the electronic charge. Since the above assumptions are consistent with those of Ref. 2, Eq. (5) is valid in the AOTV flight regimes. The relative importance of Q_{eR} to the electron-translation (*eT*) energy transfer rate considered in Ref. 2.

$$Q_{eT}=\sum_s Q_{eT,s} \tag{7}$$

where $Q_{eT,s}$ is the rate of electron energy loss due to elastic collisions with species s, is shown for N_2 and O_2. For the Maxwellian distribution f_e with the electron temperature T_e, $Q_{eT,s}$ is given by[2,9]

$$Q_{eT,s}=n_e\left(\frac{3}{2}\kappa T_e-\frac{3}{2}\kappa T\right)\Big/\tau_{eT,s} \tag{8}$$

with the *eT* relaxation time

$$\tau_{eT,s}=[(2m_e/m_s)\nu_{es}^*]^{-1} \tag{9}$$

where ν_{es}^* is the electron collision frequency defined by [Eq. (7) of Ref. 2 should be multiplied by m_e]

$$\nu_{es}^*=(n_s m_e/3n_e\kappa T_e)\int\sigma_{es}c_e'^3 f_e \mathrm{d}c_e' \tag{10}$$

m_s the mass of species s, and σ_{es} the momentum-transfer cross section of species s. For ν_{es}^* evaluated[2] for the constant σ_{es}

$$\nu_{es}^*=(4/3)n_s\sigma_{es}(8\kappa T_e/\pi m_e)^{1/2} \tag{11}$$

[the factor of 4/3 being disregarded in Eq. (64) of Ref. 2], the ratio of $Q_{eR,s}/Q_{eT,s}$ is reduced to

$$\begin{aligned}Q_{eR,s}/Q_{eT,s}&=\tau_{eT,s}/\tau_{eR,s}\\&=(m_s/m_e)(\sigma_{R,s}/\sigma_{es})(\theta_{R,s}/T_e)\end{aligned} \tag{12}$$

Using the molecular values[6,8] of $\theta_{R,N_2}=2.86$ K and $\theta_{R,O_2}=2.07$ K, $\sigma_{R,N_2}=0.51$ Å^2 and $\sigma_{R,O_2}=1.5$ Å^2 ($Q_{N_2}=1.04$ ea_0^2 and $Q_{O_2}=1.8$ ea_0^2), and $\sigma_{es}\lesssim 10$ Å^2, Eq. (12) leads to

$$Q_{eR,s}/Q_{eT,s}\gtrsim 10^4/T_e \tag{13}$$

which indicates that the *eR* energy transfer exceeds the *eT* one at $T_e\lesssim 10^4$ K and becomes dominant at $T_e\lesssim 10^3$ K, considering that the threshold energy for vibrational excitations

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is 0.289 (N_2) and 0.193 (O_2) electron volts. It is noted that the electron energy gain rate $-Q_{eR}$ [Eqs. (1), (5), and (6)] should be added to the right-hand side term of the electron energy conservation equation of Ref. 2 [Eqs. (16), (36), (37), and (79)].

$Q_{eV,s}$ is also represented[10] by Eq. (3), where i and j denote vibrational states. For the Boltzmann distribution n_i with the vibrational temperature T_v, and the Maxwellian distribution f_e with the electron temperature T_e, which yields the detailed balancing

$$k_{ji}/k_{ij} = \exp(\Delta E_{ji}/\kappa T_e) \tag{14}$$

$Q_{eV,s}$ is reduced to

$$Q_{eV,s} = n_e \sum_i n_i \sum_j k_{i,i+j}\beta_j \tag{15}$$

where

$$n_i = n_s[1-\exp(-\theta_{v,s}/T_v)]\exp(-i\theta_{v,s}/T_v) \tag{16}$$

$$\beta_j = j\kappa\theta_{v,s}\{1-\exp[j\theta_{v,s}(1/T_e - 1/T_v)]\} \tag{17}$$

and $\theta_{v,s}$ is the vibrational characteristic temperature. The theoretical data[11] of $\sigma_{i,i+j}$ indicate 1) a complicated dependence on the electron energy and 2) the evaluation of the summation in Eq. (15) requires several terms for i $(=0,1,2,\ldots)$ and a few terms for j $(=1,2,\ldots)$.

Although Eq. (15), in general, may not be reduced to a simpler expression, it is pointed out that, in a particular case, it leads to the same Landau-Teller type expression as derived in Ref. 2. When the vibrational transition rate coefficient k_{ij} satisfies[12]

$$k_{i,i+j} \cong k_{0j}, \qquad \mathrm{i} \geq 1 \tag{18}$$

or

$$n_i k_{i,i+j} \ll n_0 k_{0j}, \qquad i \geq 1 \tag{19}$$

Eq. (15) is reduced, respectively, to

$$Q_{eV,s} = n_e n_s \sum_j k_{0j}\beta_j \tag{20}$$

$$Q_{eV,s} = n_e n_s [1-\exp(-\theta_{v,s}/T_v)] \sum_j k_{0j}\beta_j \tag{21}$$

At $T_e \cong T_v \gg \theta_{v.s}$, $\beta_j = \kappa(j\theta_{v,s})^2(1/T_v - 1/T_e)$ and Eqs. (20) and (21) lead to the Landau-Teller type expression[2]

$$Q_{eV,s} = \rho_s(e^{**}_{v,s} - e_{v,s})/\tau_{eV,s} \tag{22}$$

with the eV relaxation time

$$\tau_{eV,s} = [2K_0(\theta_{v,s}/T_e)^\lambda]^{-1} \tag{23}$$

where $\rho_s(=m_s n_s)$ is the mass density; $e_{v,s}(\cong \kappa T_v/m_s)$ and $e^{**}_{v,s}$ $(\cong \kappa T_e/m_s)$ are the vibrational and equilibrium $(T_v = T_e)$ energies[2]; $\lambda = 2$ and 3 for Eqs. (20) and (21), respectively; and $K_0 = (n_e/2)\Sigma_j k_{0j} j^2$ is defined in Ref. 2. It is noted that $\tau_{eV,s}$ for $\lambda = 2$ is half of Lee's relaxation time [Eq. (A17) of Ref. 2].

References

[1] Nelson, H. F. (ed), "Thermal Design of Aeroassisted Orbital Transfer Vehicles," *Progress in Astronautics and Aeronautics*, Vol. 96, AIAA, New York, 1985.

[2] Lee, J.-H., "Basic Governing Equations for the Flight Regimes of Aeroassisted Orbital Transfer Vehicles," *Progress in Astronautics and Aeronautics*, Vol. 96, edited by H. F. Nelson, AIAA, New York, 1985, pp. 3-53.

[3] Sutton, G. W. and Sherman, A., *Engineering Magnetohydrodynamics*, McGraw-Hill, New York, 1965.

[4] Gerjuoy, E. and Stein, S., "Rotational Excitation by Slow Electrons," *Physical Review*, Vol. 97, March 1955, pp. 1671-1679.

[5] Mentzoni, M. H. and Row, R. V., "Rotational Excitation and Electron Relaxation in Nitrogen," *Physical Review*, Vol. 130, June 1963, pp. 2312-2316.

[6] Engelhardt, A. G., Phelps, A. V., and Risk, C. G., "Determination of Momentum Transfer and Inelastic Collision Cross Sections for Electrons in Nitrogen Using Transport Coefficients," *Physical Review*, Vol. 135, Sept. 1964, pp. A1566-A1574.

[7] Mentzoni, M. H. and Narasinga Rao, K. V., "Electron Energy Relaxation in Oxygen," *Physical Review Letters*, Vol. 14, May 1965, pp. 779-780.

[8] Hake, R. D. Jr. and Phelps, A. V., "Momentum-Transfer and Inelastic-Collision Cross Sections for Electrons in O_2, CO, and CO_2," *Physical Review*, Vol. 158, June 1967, pp. 70-84.

[9] Koura, K., "Monte Carlo Simulation of Electron Thermalization in Gases. III. Epithermal Electrons in Molecular Nitrogen," *Journal of Chemical Physics*, Vol. 81, July 1984, pp. 303-308.

[10] Koura, K., "Nonequilibrium Electron Velocity Distribution and Temperature in Thermalization of Low-Energy Electrons in Molecular Hydrogen," *Journal of Chemical Physics*, Vol. 79, Oct. 1983, pp. 3367-3372.

[11] Chandra, N. and Temkin, A., "Tabulation of Hybrid Theory Calculated e-N_2 Vibrational and Rotational Cross Sections," NASA TN-D-8347, Washington, DC, Oct. 1976.

[12] Capitelli, M., Dilonardo, M., and Gorse, C., "Electron Energy Distribution Functions and Dissociation Kinetics of HCl in Nonequilibrium Plasmas," *Chemical Physics*, Vol. 43, Nov. 1979, pp. 403-414.

Navier-Stokes Similarity Solution for the Planar Liquid Wall Jet

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Introduction

THE laminar liquid wall jet provides a simple example of a boundary-layer type flow. In the absence of gravitational or surface tension effects, Watson[1] found the boundary-layer similarity solutions for both the radial and planar cases. In the plane jet far field the flow may be considered to have emanated from a source of given strength. The velocity is directed along rays and its distribution is independent of the distance from the source. The Reynolds number for the problem is independent of distance (e.g., volume flow/viscosity) and the solution of Watson is valid when the Reynolds number is large. In this note, for distances large with respect to the jet exit dimension, a similar solution for the plane liquid wall jet, valid for all Reynolds numbers, is deduced. The solution reduces to the boundary-layer result[1] for large Reynolds number and includes a description for the wall jet flow directed toward a sink. Furthermore, it is shown that these solutions are special cases of the Navier-Stokes result for nonparallel channel flows uncovered by Jeffery[2] and Hamel[3] and elucidated by several investigators.[4-7]

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